Richmond Public Schools

Curriculum Framework

Grade 7 Honors (7/8)

Strand: Measurement and Geometry

8.16 The student will

a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero;

b) identify the slope and y-intercept of a linear function given a table of values, a graph, or an equation in $y = mx + b$ form;

c) determine the independent and dependent variable, given a practical situation modeled by a linear function;

d) graph a linear function given the equation in $y = mx + b$ form; and

e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs.

7.10 The student will

a) determine the slope, m, as rate of change in a proportional relationship between two quantities and write an equation in the form $y = mx$ to represent the relationship;

b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in $y = mx$ form where m represents the slope as rate of change.

c) determine the y-intercept, b, in an additive relationship between two quantities and write an equation in the form $y = x + b$ to represent the relationship;

d) graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form $y = x + b$, where b represents the y-intercept; and

e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.

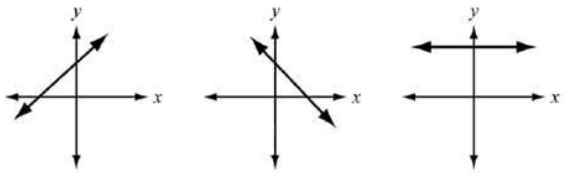
Suggested Pacing

Related Standards

Spiral Down:	Spiral Up: Algebra: • SOL A.6a,c Geometry: • SOL G.3b

Essential Questions	Common Misconceptions
• What are some strategies used for graphing a linear equation? • What does the rate of change and y-intercept represent in a real-life world situation? • How are proportional relationships related to rate of change(slope)?	• Slope(rise/run): student may reverse the ratio (change in x/change in y) • Graphing: students may follow the wrong process for graphing the equation using the slope and y-intercept • Special Slopes: students may have trouble remembering the direction of lines with slopes that are negative, positive, and zero.

Understanding the Standard	Essential Knowledge and Skills
• SOL 8.16: • A linear function is an equation in two variables whose graph is a straight line, a type of continuous function. • A linear function represents a situation with a constant rate. For example, when driving at a rate of 35 mph, the distance increases as the time increases, but the rate of speed remains the same. • Slope (m) represents the rate of change in a linear function or the "steepness" of the line. The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change. – slope = $\frac{\text{change in } y}{\text{change in } x} = \frac{\text{vertical change}}{\text{horizontal change}}$ • A line is increasing if it rises from left to right. The slope is positive (i.e., $m > 0$). • A line is decreasing if it falls from left to right. The slope is negative (i.e., $m < 0$). • A horizontal line has zero slope (i.e., $m = 0$).	• SOL 8.16: • Recognize and describe a line with a slope that is positive, negative, or zero (0). (a) • Given a table of values for a linear function, identify the slope and y-intercept. The table will include the coordinate of the y-intercept. (b) • Given a linear function in the form $y = mx + b$, identify the slope and y-intercept. (b) • Given the graph of a linear function, identify the slope and y-intercept. The value of the y-intercept will be limited to integers. The coordinates of the ordered pairs shown in the graph will be limited to integers. (b) • Identify the dependent and independent variable, given a practical situation modeled by a linear function. (c) • Given the equation of a linear function in the form $y = mx + b$, graph the function. The value of the y-intercept will be limited to integers. (d) • Write the equation of a linear function in the form $y = mx + b$ given values for the slope, m, and the y-intercept or given a practical situation in which the slope, m, and y-intercept are described verbally.(e) • Make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs. (e).



A line with a ***positive slope*** slants up to the right.

A line with a ***negative slope*** slants down to the right.

A line with a ***slope of 0*** is horizontal.

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- A discussion about lines with undefined slope (vertical lines) should occur with students in grade eight mathematics to compare undefined slope to lines with a defined slope. Further exploration of this concept will occur in Algebra I.
- A linear function can be written in the form $y = mx + b$, where m represents the slope or rate of change in y compared to x, and b represents the y-intercept of the graph of the linear function. The y-intercept is the point at which the graph of the function intersects the y-axis and may be given as a single value, b, or as the location of a point $(0, b)$.
  - Example: Given the equation of the linear function $y = -3x + 2$, the slope is -3 or $\frac{-3}{1}$ and the y-intercept is 2 or (0, 2).
  - Example: The table of values represents a linear function.

In the table, the point (0, 2) represents the y-intercept. The slope is determined by observing the change in each y-value compared to the corresponding change in the x-value.

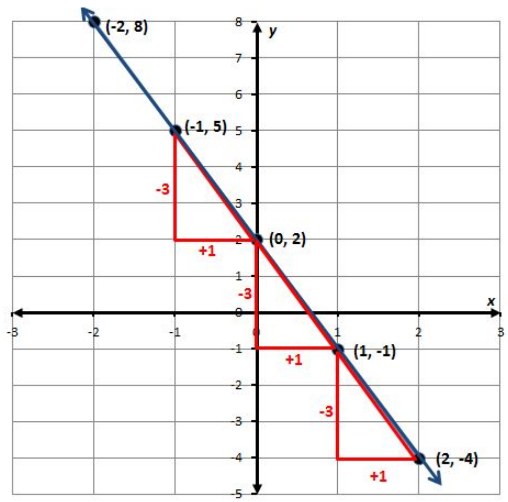
change in x	x	y	change in y
+1	-2	8	-3
+1	-1	5	-3
+1	0	2	-3
+1	1	-1	-3
	2	-4	

- SOL 7.10:
- Determine the slope, m, as rate of change in a proportional relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = mx$ to represent the relationship. Slope will be limited to positive values. (a)
- Graph a line representing a proportional relationship, between two quantities given an ordered pair on the line and the slope, m, as rate of change. Slope will be limited to positive values. (b)
- Graph a line representing a proportional relationship between two quantities given the equation of the line in the form $y = mx$, where m represents the slope as rate of change. Slope will be limited to positive values. (b)
- Determine the y-intercept, b, in an additive relationship between two quantities given a table of values or a verbal description, including those represented in a practical situation, and write an equation in the form $y = x + b$, $b \neq 0$, to represent the relationship. (c)
- Graph a line representing an additive relationship ($y = x + b$, $b \neq 0$) between two quantities, given an ordered pair on the line and the y-intercept (b). The y-intercept (b) is limited to integer values and slope is limited to 1. (d)
- Graph a line representing an additive relationship between two quantities, given the equation in the form $y = x + b$, $b \neq 0$. The y-intercept (b) is limited to integer values and slope is limited to 1. (d)
- Make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs. (e)

slope = $m = \frac{\textit{change in y-value}}{\textit{change in x-value}} = \frac{-3}{+1} = -3$

- The slope, m, and y-intercept of a linear function can be determined given the graph of the function.

  – Example: Given the graph of the linear function, determine the slope and y-intercept.



Given the graph of a linear function, the y-intercept is found by determining where the line intersects the y-axis. The y-intercept would be 2 or located at the point (0, 2). The slope can be found by determining the change in each y-value compared to the change in each x-value. Here, we could use slope triangles to help visualize this:

slope = $m = \frac{\textit{change in y-value}}{\textit{change in x-value}} = \frac{-3}{+1} = -3$

- Graphing a linear function given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to determine points on the line.

– Example: Graph the linear function whose equation is $y = 5x - 1$.

In order to graph the linear function, we can create a table of values by substituting arbitrary values for x to determining coordinating values for y:

x	$5x - 1$	y
-1	5(-1) - 1	-6
0	5(0) - 1	-1
1	5(1) - 1	4
2	5(2) - 1	9

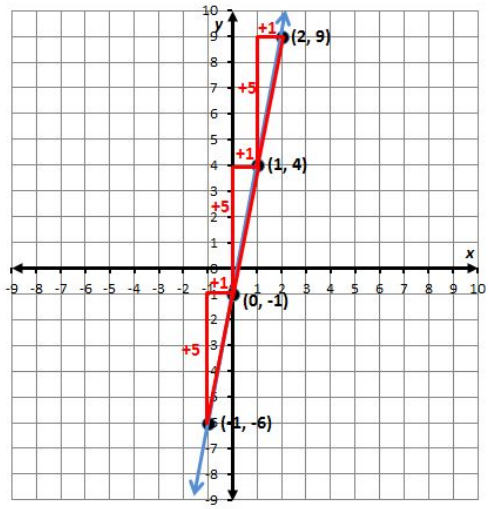
The values can then be plotted as points on a graph.

Knowing the equation of a linear function written in $y = mx + b$ provides information about the slope and y-intercept of the function. If the equation is $y = 5x - 1$, then the slope, m, of the line is 5 or $\frac{5}{1}$ and the y-intercept is -1 and can be located at the point $(0, -1)$. We can graph the line by first plotting the y-intercept. We also know,

$$\text{slope} = m = \frac{\text{change in } y\text{-value}}{\text{change in } x\text{-value}} = \frac{+5}{+1}$$

Other points can be plotted on the graph using the relationship between the y and x values.

Slope triangles can be used to help locate the other points as shown in the graph below:



- A table of values can be used in conjunction with using slope triangles to verify the graph of a linear function. The *y*-intercept is located on the *y*-axis which is where the *x*-coordinate is 0. The change in each *y*-value compared to the corresponding *x*-value can be verified by the patterns in the table of values.

x	*y*
-1	-6
0	-1
1	4
2	9

- The axes of a coordinate plane are generally labeled *x* and *y*; however, any letters may be used that are appropriate for the function.
- A function has values that represent the input (*x*) and values that represent the output (*y*). The independent variable is the input value.
- The dependent variable depends on the independent variable and is the output value.

- Below is a table of values for finding the approximate circumference of circles, $C = \pi d$, where the value of π is approximated as 3.14.

Diameter	Circumference
1 in.	3.14 in.
2 in.	6.28 in.
3 in.	9.42 in.
4 in.	12.56 in.

  - The independent variable, or input, is the diameter of the circle. The values for the diameter make up the domain.
  - The dependent variable, or output, is the circumference of the circle. The set of values for the circumference makes up the range.

- In a graph of a continuous function every point in the domain can be interpreted. Therefore, it is possible to connect the points on the graph with a continuous line because every point on the line answers the original question being asked.

- The context of a problem may determine whether it is appropriate for ordered pairs representing a linear relationship to be connected by a straight line. If the independent variable (x) represents a discrete quantity (e.g., number of people, number of tickets, etc.) then it is not appropriate to connect the ordered pairs with a straight line when graphing. If the independent variable (x) represents a continuous quantity (e.g., amount of time, temperature, etc.), then it is appropriate to connect the ordered pairs with a straight line when graphing.
  - Example: The function $y = 7x$ represents the cost in dollars (y) for x tickets to an event. The domain of this function would be discrete and would be represented by discrete points on a graph. Not all values for x could be represented and connecting the points would not be appropriate.

- Example: The function $y = -2.5x + 20$ represents the number of gallons of water (y) remaining in a 20-gallon tank being drained for x number of minutes. The domain in this function would be continuous. There would be an x-value representing any point in time until the tank is drained so connecting the points to form a straight line would be appropriate (Note: the context of the problem limits the values that x can represent to positive values, since time cannot be negative.).

- Functions can be represented as ordered pairs, tables, graphs, equations, physical models, or in words. Any given relationship can be represented using multiple representations.
- The equation $y = mx + b$ defines a linear function whose graph (solution) is a straight line. The equation of a linear function can be determined given the slope, m, and the y-intercept, b. Verbal descriptions of practical situations that can be modeled by a linear function can also be represented using an equation.
  - Example: Write the equation of a linear function whose slope is $\frac{3}{4}$ and y-intercept is −4, or located at the point (0, −4).
  - The equation of this line can be found by substituting the values for the slope, $m = \frac{3}{4}$, and the y-intercept, $b = -4$, into the general form of a linear function $y = mx + b$. Thus, the equation would be $y = \frac{3}{4}x - 4$.
  - Example: John charges a $30 flat fee to trouble shoot a personal watercraft that is not working properly and $50 per hour needed for any repairs. Write a linear function that represents the total cost, y of a personal watercraft repair, based on the number of hours, x, needed to repair it. Assume that there is no additional charge for parts.
  - In this practical situation, the y-intercept, b, would be $30, to represent the initial flat fee to trouble shoot the watercraft. The slope, m, would be $50, since that would represent the rate per hour. The equation to represent this situation would be $y = 50x + 30$.
- A proportional relationship between two variables can be represented by a linear function $y = mx$ that passes through the point (0, 0) and thus has a

y-intercept of 0. The variable *y* results from *x* being multiplied by *m,* the rate of change or slope.

- The linear function $y = x + b$ represents a linear function that is a non-proportional additive relationship. The variable *y* results from the value *b* being added to *x.* In this linear relationship, there is a y-intercept of *b,* and the constant rate of change or slope would be 1. In a linear function with a slope other than 1, there is a coefficient in front of the *x* term, which represents the constant rate of change, or slope.
- Proportional relationships and additive relationships between two quantities are special cases of linear functions that are discussed in grade seven mathematics.
- SOL 7.10:
- When two quantities, *x* and *y*, vary in such a way that one of them is a constant multiple of the other, the two quantities are “proportional”. A model for that situation is $y = mx$ where *m* is the slope or rate of change. Slope may also represent the unit rate of a proportional relationship between two quantities, also referred to as the constant of proportionality or the constant ratio of *y* to *x*.
- The slope of a proportional relationship can be determined by finding the unit rate.

mple: The ordered pairs (4, 2) and (6, 3) make up points that could be included on the graph of a proportional relationship. Determine the slope, or rate of change, of a line passing through these points. Write an equation of the line representing this proportional relationship.

slope, or rate of change, would be $\frac{1}{2}$ or 0.5 since the *y*-coordinate of each ordered pair would result by multiplying $\frac{1}{2}$ times the *x*-coordinate. This would also be the unit rate of this proportional relationship. The ratio of *y* to *x* is the same for each ordered pair. That is, $\frac{y}{x} = \frac{2}{4} = \frac{3}{6} = \frac{1}{2} = 0.5$

equation of a line representing this proportional relationship of y to x is $y = \frac{1}{2}x$ or $y = 0.5x$.

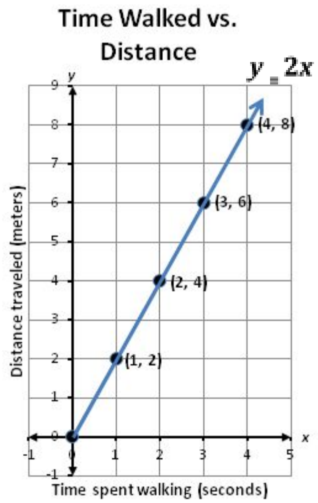
- The slope of a line is a rate of change, a ratio describing the vertical change to the horizontal change of the line.

$$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{\text{vertical change}}{\text{horizontal change}}$$

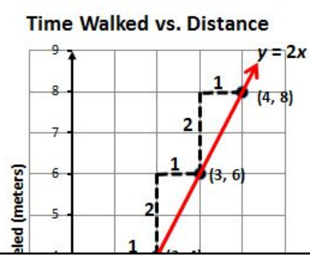
- The graph of the line representing a proportional relationship will include the origin (0, 0).
- A proportional relationship between two quantities can be modeled given a practical situation. Representations may include verbal descriptions, tables, equations, or graphs. Students may benefit from an informal discussion about independent and dependent variables when modeling practical situations. Grade eight mathematics formally addresses identifying dependent and independent variables.
  - Example (using a table of values): Cecil walks 2 meters every second (verbal description). If x represents the number of seconds and y represents the number of meters he walks, this proportional relationship can be represented using a table of values:

econds)				
neters)				

  - This proportional relationship could be represented using the equation $y = 2x$, since he walks 2 meters for each second of time. That is, $\frac{y}{x} = \frac{2}{1} = \frac{4}{2} = \frac{6}{3} = \frac{8}{4} = 2$, the unit rate (constant of proportionality) is 2 or $\frac{2}{1}$. The same constant ratio of y to x exists for every ordered pair. This proportional relationship could be represented by the following graph:



- A graph of a proportional relationship can be created by graphing ordered pairs generated in a table of values (as shown above), or by observing the rate of change or slope of the relationship and using slope triangles to graph ordered pairs that satisfy the relationship given.
  - Example (using slope triangles): Cecil walks 2 meters every second. If x represents the number of seconds and y represents the number of meters he walks, this proportional relationship can be represented graphically using slope triangles.



The rate of change from (1, 2) to (2, 4) is 2 units up (the change in *y*) and 1 unit to the right
(the change in *x*), $\frac{2}{1}$ or 2. Thus, the slope of this line is 2. Slope triangles can be used to generate points on a graph that satisfy this relationship.

- Proportional thinking requires students to thinking multiplicatively. However, the relationship between two quantities is not always proportional. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, since context can help students to see the relationship.
  - Example:

Additive relationship:

x	*y*
2 (+8 →)	10
3 (+8 →)	11
4 (+8 →)	12
5 (+8 →)	13

Multiplicative relationship:

x	*y*
2 (·5 →)	10
3 (·5 →)	15
4 (·5 →)	20
5 (·5 →)	25

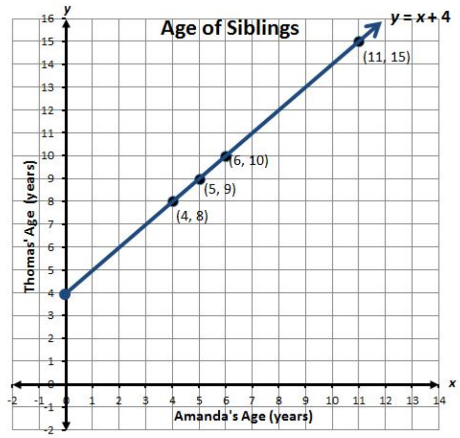
In the additive relationship, *y* is the result of adding 8 to *x*.
In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.

The ordered pair (2, 10) is a quantity in both relationships, however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- Two quantities, x and y, have an additive relationship when a constant value, b, exists where $y = x + b$, where $b \neq 0$. An additive relationship is not proportional and its graph does not pass through (0, 0). Note that b can be a positive value or a negative value. When b is negative, the right side of the equation could be written using a subtraction symbol (e.g., if b is −5, then the equation $y = x - 5$ could be used).
    - Example: Thomas is four years older than his sister, Amanda (verbal description). The following table shows the relationship between their ages at given points in time.

Amanda's Age	4 (+4)	5 (+4)	6 (+4)	11 (+4)
Thomas' Age	8	9	10	15

The equation that represents the relationship between Thomas' age and Amanda's age is $y = x + 4$. A graph of the relationship between their ages is shown below:



- Graphing a line given an equation can be addressed using different methods. One method involves determining a table of ordered pairs by substituting into the equation values for one variable and solving for the other variable, plotting the ordered pairs in the coordinate plane, and connecting the points to form a straight line. Another method involves using slope triangles to determine points on the line.
  - Example: Graph the equation $y = x - 1$.

In order to graph the equation, we can create a table of values by substituting arbitrary values for x to determine coordinating values for y:

x	x - 1	y
-1	(-1) - 1	-2
0	(0) - 1	-1
1	(1) - 1	0
2	(2) - 1	1

Thes s the points ( ) on a graph.

An equation written in $y = x + b$ form provides information about the graph. If the equation is $y = x - 1$, then the slope, m, of the line is 1 or $\frac{1}{1}$ and the point where the line crosses the y-axis can be located at (0, -1). We also know, slope = $m = \frac{change\ in\ y-value}{change\ in\ x-value} = \frac{+1}{+1}\ or\ \frac{-1}{-1}$

So we can plot some other points on the graph using this relationship between *y* and *x* values.

A table of values can be used to determine the graph of a line. The *y*-intercept is located on the *y*-axis which is where the *x*-coordinate is 0. The change in each *y*-value compared to the corresponding *x*-value can be verified by the patterns in the table of values.

	x
	-1
+1	0
+1	1
+1	2

Vocabulary

SOL 8.16:

Linear Function	Slope(m)	Y-Intercept (b)
Independent Variable (x)	Dependent Variable (y)	proportion

SOL 7.10:

Slope	y-intercept	Linear Function
Function	Relation	Ordered Pair

Instructional Activities Organized by Learning Objective

Textbook

Notes

Resources

- Print
- Technology-based

Station Activities

Assessment

Cross-Curricular Connections

Tiered Differentiations