Richmond Public Schools
Curriculum Framework
Grade 6 Honors (6/7)

Strand: Measurement and Geometry

6.12 The student will
a) represent a proportional relationship between two quantities, including those arising from practical situations;
b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;
c) determine whether a proportional relationship exists between two quantities; and
d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.

Suggested Pacing

Related Standards

Spiral Down:	Spiral Up:
4th Grade:How do I represent and/or solve real-world and mathematical problems using rates, ratios, and/or percents? ● SOL 4.15 5th Grade: ● SOL 5.18 ● SOL 5.19a	7th Grade: ● SOL 7.10e 8th Grade: ● SOL 8.15 ● SOL 8.16 e

Essential Questions	Common Misconceptions
● How does a ratio help us to compare quantities? ● How does multiplication and division help us to understand ratios concepts and apply it to problem solving? ● What kinds of relationships can proportions represent? ● Why are ratios and proportions important? ● How are they used in the real world?	● Setting up the ratio/proportion: students put the quantities in the wrong location; students do not line the information up correctly ● Solving a proportion: Students multiply the wrong quantities

Understanding the Standard	Essential Knowledge and Skills
● A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities.	● Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a ratio. (a)

- Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio.

- A proportional relationship consists of two quantities where there exists a constant number (constant of proportionality) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity.

- Proportional thinking requires students to thinking multiplicatively, versus additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, because context can help students to see the relationship. Students will explore algebraic representations of additive relationships in grade seven.

  - Example:

  o In the additive relationship, y is the result of adding 8 to x.

  o In the multiplicative relationship, y is the result of multiplying 5 times x.

  o The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- Make a table of equivalent ratios to represent a proportional relationship between two quantities, when given a practical situation. (a)

- Identify the unit rate of a proportional relationship represented by a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (b)

- Determine a missing value in a ratio table that represents a proportional relationship between two quantities using a unit rate. Unit rates are limited to positive values. (b)

- Determine whether a proportional relationship exists between two quantities, when given a table of values or a verbal description, including those represented in a practical situation. Unit rates are limited to positive values. (c)

- Determine whether a proportional relationship exists between two quantities given a graph of ordered pairs. Unit rates are limited to positive values. (c)

- Make connections between and among multiple representations of the same proportional relationship using verbal descriptions, ratio tables, and graphs. Unit rates are limited to positive values. (d)

- Students have had experiences with tables of values (input/output tables that are additive and multiplicative) in elementary grades.

- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table.

  - Example: Given that the ratio of y to x in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.

Ratio that is given

dents have had experience with tables of values (input/output tables) in elementary grades and the concept of a ratio table should be connected to their prior knowledge of representing number patterns in tables.

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).

- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.

  - Example: If it costs $10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be $2.00/per item (a ratio of 2:1 comparing cost to number of items).

st in $ (*y*)	.00	00	).00	0.00

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator. Example: It costs $8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?

$$\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$$

it would cost $0.50 per cookie, which would be the unit rate.

- Example: $\frac{8}{16}$ and 40 to 10 are ratios, but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are unit rates.

Students in grade six should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher

- grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in grade six.

- Example of a proportional relationship:

Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This ratio table represents the cost (*y*) per number of pizzas ordered (*x*).

x number of pizzas	1	2	3	4
y total cost	8	16	24	32

In this relationship, the ratio of *y* (cost in $) to *x* (number of pizzas) in each ordered pair is the same:

$$\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$$

- Example of a non-proportional relationship:

Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

The ratios represented in the table above are not equivalent.

In this relationship, the ratio of y to x in each ordered pair is not the same:

$$\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$$

Other non-proportional relationships will be studied in later mathematics courses.

- Proportional relationships can be described verbally using the phrases "for each," "for every," and "per."

- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (x, y) that represent pairs of values that may be represented in a ratio table.

- Proportional relationships can be expressed using verbal descriptions, tables, and graphs.

  - Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If x represents how many liters of syrup are in the mixture and y represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:
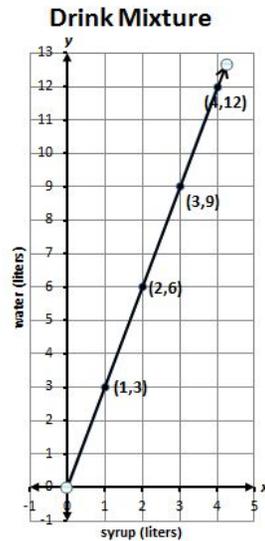
Syrup (liters) x	1	2	3	4
Water (liters) y	3	6	9	12

The ratio of the amount of water (y) to the amount of syrup (x) is 3:1.
Additionally, the proportional relationship may be graphed using the ordered pairs in the table.

Drink Mixture

- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared.

  - Example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:

Water (liters) x	3	6	9	12
Syrup (liters) y	1	2	3	4

In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.

The graph of this relationship could be represented by:

Drink Mixture
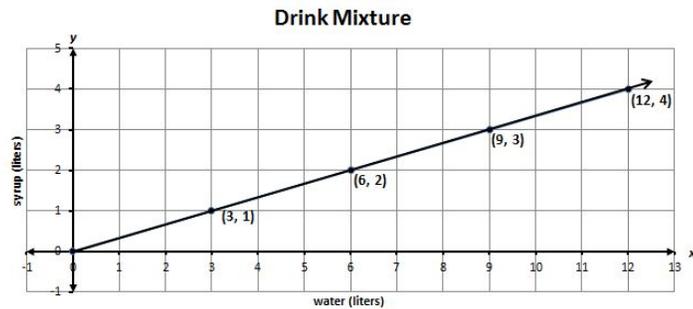
Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph.

● Double number line diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios.

- Example:
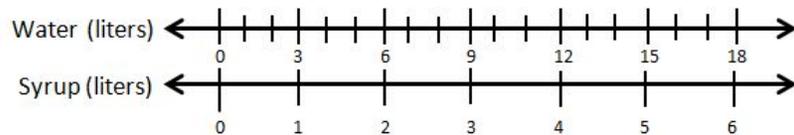
In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.

● A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through (0, 0), creating a pattern of horizontal and vertical increases. The context of the problem and the type of data being represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph.
- Example of the graph of a non-proportional relationship:

Time vs. Distance
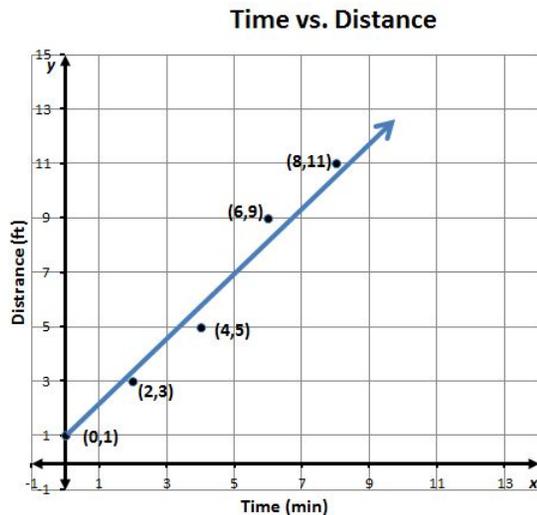
The relationship of distance (y) to time (x) is non-proportional. The ratio of y to x for each ordered pair is not equivalent. That is,

$$\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$$

The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point (0, 0), thus the relationship of y to x cannot be considered proportional.

- Practical situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most cases the values for x and y are not negative.

- Unit rates are not typically negative in practical situations involving proportional relationships.

- A unit rate could be used to find missing values in a ratio table.

  - Example: A store advertises a price of $25 for 5 DVDs. What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

DVDs	1	2	3	4	5
Cost	5	?	?	?	25

The ratio of $25 per 5 DVDs is also equivalent to a ratio of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.

Vocabulary	Instructional Activities Organized by Learning Objective

SOL 6.12

Textbook

Ratio	Proportion	Unit Rate
Rate	Ratio Table	Proportional Relationship

Notes

Resources
- Print
- Technology-based

Assessment

Station Activities

Cross-Curricular Connections	Tiered Differentiations