Richmond Public Schools

Curriculum Framework

Grade 6

Strand: Measurement and Geometry	
6.12 The student will **a) represent a proportional relationship between two quantities, including those arising from practical situations;** **b) determine the unit rate of a proportional relationship and use it to find a missing value in a ratio table;** **c) determine whether a proportional relationship exists between two quantities; and** **d) make connections between and among representations of a proportional relationship between two quantities using verbal descriptions, ratio tables, and graphs.**	
Suggested Pacing	
Second Nine Weeks-12 Instructional Days	
Related Standards	
6.1 The student will represent relationships between quantities using ratios, and will use appropriate notations, such as $\frac{a}{b}$, *a* to *b*, and *a*:*b*. **7.10** The student will a) determine the slope, m, as rate of change in a proportional relationship between two quantities and write an equation in the form y = mx to represent the relationship; b) graph a line representing a proportional relationship between two quantities given the slope and an ordered pair, or given the equation in y = mx form where m represents the slope as rate of change; c) determine the y-intercept, b, in an additive relationship between two quantities and write an equation in the form y = x + b to represent the relationship; d) graph a line representing an additive relationship between two quantities given the y-intercept and an ordered pair, or given the equation in the form y = x + b, where b represents the y-intercept; and e) make connections between and among representations of a proportional or additive relationship between two quantities using verbal descriptions, tables, equations, and graphs.	**8.16** The student will a) recognize and describe the graph of a linear function with a slope that is positive, negative, or zero; b) identify the slope and y-intercept of a linear function, given a table of values, a graph, or an equation in y = mx + b form; c) determine the independent and dependent variable, given a practical situation modeled by a linear function; d) graph a linear function given the equation in y = mx + b form; and e) make connections between and among representations of a linear function using verbal descriptions, tables, equations, and graphs. **A.6** The student will a) determine the slope of a line when given an equation of the line, the graph of the line, or two points on the line; b) write the equation of a line when given the graph of the line, two points on the line, or the slope and a point on the line; and c) graph linear equations in two variables.
Essential Questions	**Common Misconceptions**

Understanding the Standard	Essential Knowledge and Skills
● A ratio is a comparison of any two quantities. A ratio is used to represent relationships within a quantity and between quantities. ● Equivalent ratios arise by multiplying each value in a ratio by the same constant value. For example, the ratio of 4:2 would be equivalent to the ratio 8:4, since each value in the first ratio could be multiplied by 2 to obtain the second ratio. ● A proportional relationship consists of two quantities where there exists a constant number (constant of proportionality) such that each measure in the first quantity multiplied by this constant gives the corresponding measure in the second quantity. ● Proportional thinking requires students to thinking multiplicatively, versus additively. The relationship between two quantities could be additive (i.e., one quantity is a result of adding a value to the other quantity) or multiplicative (i.e.., one quantity is the result of multiplying the other quantity by a value). Therefore, it is important to use practical situations to model proportional relationships, because context can help students to see the relationship. Students will explore algebraic representations of additive relationships in grade seven. ○ Example:	**Essential Questions** **6.12a** ● What is the relationship between a ratio and a proportion? ● How does a proportion compare to two equivalent ratios? ● How would you use a table to show a proportional relationship? ● What is a situation that demonstrates a proportional relationship? ● What is a situation that does not demonstrate a proportional relationship? **6.12b** ● When would a unit rate be useful to solve a problem? ● How do you interpret a unit rate? ● What are the similarities between simplifying a fraction and showing proportional relationship? **6.12c** ● What is a proportional relationship? **6.12d** ● How is a proportional relationship, when written in words, shown in a ratio table and a graph?

Additive relationship:

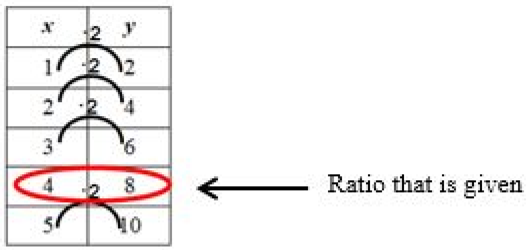
x	y
2 $\xrightarrow{+8}$	10
3 $\xrightarrow{+8}$	11
4 $\xrightarrow{+8}$	12
5 $\xrightarrow{+8}$	13

Multiplicative relationship:

x	y
2 $\xrightarrow{\cdot 5}$	10
3 $\xrightarrow{\cdot 5}$	15
4 $\xrightarrow{\cdot 5}$	20
5 $\xrightarrow{\cdot 5}$	25

- ○ In the additive relationship, *y* is the result of adding 8 to *x.*
- ○ In the multiplicative relationship, *y* is the result of multiplying 5 times *x*.
- ○ The ordered pair (2, 10) is a quantity in both relationships; however, the relationship evident between the other quantities in the table, discerns between additive or multiplicative.

- Students have had experiences with tables of values (input/output tables that are additive and multiplicative) in elementary grades.
- A ratio table is a table of values representing a proportional relationship that includes pairs of values that represent equivalent rates or ratios. A constant exists that can be multiplied by the measure of one quantity to get the measure of the other quantity for every ratio pair. The same proportional relationship exists between each pair of quantities in a ratio table.
    - ○ Example: Given that the ratio of *y* to *x* in a proportional relationship is 8:4, create a ratio table that includes three additional equivalent ratios.



Students have had experience with tables of values (input/output tables) in elementary grades and the concept of a ratio table should be connected to their prior knowledge of representing number patterns in tables.

- A rate is a ratio that involves two different units and how they relate to each other. Relationships between two units of measure are also rates (e.g., inches per foot).
- A unit rate describes how many units of the first quantity of a ratio correspond to one unit of the second quantity.
    - Example: If it costs $10 for 5 items at a store (a ratio of 10:5 comparing cost to the number of items), then the unit rate would be $2.00/per item (a ratio of 2:1 comparing cost to number of items).

Unit Rate (1, $2.00) — Given ratio (5, $10.00)

# of items (*x*)	1	2	5	10
Cost in $ (*y*)	$2.00	$4.00	$10.00	$20.00

- Any ratio can be converted into a unit rate by writing the ratio as a fraction and then dividing the numerator and denominator each by the value of the denominator. Example: It costs $8 for 16 gourmet cookies at a bake sale. What is the price per cookie (unit rate) represented by this situation?

$$\frac{8}{16} = \frac{8 \div 16}{16 \div 16} = \frac{0.5}{1}$$

So, it would cost $0.50 per cookie, which would be the unit rate.

- Example: $\frac{8}{16}$ and 40 to 10 are ratios, but are not unit rates. However, $\frac{0.5}{1}$ and 4 to 1 are unit rates.
  Students in grade six should build a conceptual understanding of proportional relationships and unit rates before moving to more abstract representations and complex computations in higher grade levels. Students are not expected to use formal calculations for slope and unit rates (e.g., slope formula) in grade six.
  - Example of a proportional relationship:
    Ms. Cochran is planning a year-end pizza party for her students. Ace Pizza offers free delivery and charges $8 for each medium pizza. This ratio table represents the cost (y) per number of pizzas ordered (x).

x number of pizzas	1	2	3	4
y total cost	8	16	24	32

x8 x8

  In this relationship, the ratio of y (cost in $) to x (number of pizzas) in each ordered pair is the same:

$$\frac{8}{1} = \frac{16}{2} = \frac{24}{3} = \frac{32}{4}$$

- Example of a non-proportional relationship:

  Uptown Pizza sells medium pizzas for $7 each but charges a $3 delivery fee per order. This table represents the cost per number of pizzas ordered.

x number of pizzas	1	2	3	4
y total cost	10	17	24	31

The ratios represented in the table above are not equivalent.
In this relationship, the ratio of *y* to *x* in each ordered pair is not the same:

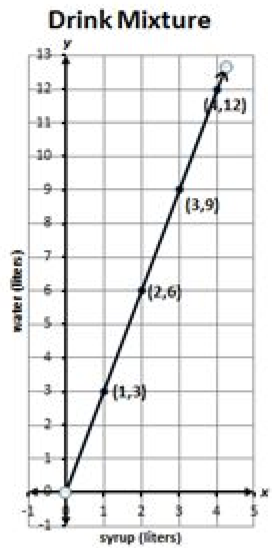
$$\frac{10}{1} \neq \frac{17}{2} \neq \frac{24}{3} \neq \frac{31}{4}$$

Other non-proportional relationships will be studied in later mathematics courses.

- Proportional relationships can be described verbally using the phrases "for each," "for every," and "per."
- Proportional relationships involve collections of pairs of equivalent ratios that may be graphed in the coordinate plane. The graph of a proportional relationship includes ordered pairs (*x*, *y*) that represent pairs of values that may be represented in a ratio table.
- Proportional relationships can be expressed using verbal descriptions, tables, and graphs.
    - Example: (verbal description) To make a drink, mix 1 liter of syrup with 3 liters of water. If *x* represents how many liters of syrup are in the mixture and *y* represents how many liters of water are in the mixture, this proportional relationship can be represented using a ratio table:

Syrup (liters) x	1	2	3	4
Water (liters) y	3	6	9	12

The ratio of the amount of water (*y*) to the amount of syrup (*x*) is 3:1. Additionally, the proportional relationship may be graphed using the ordered pairs in the table.

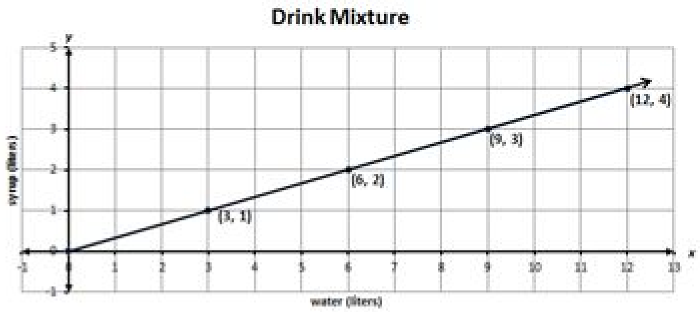


- The representation of the ratio between two quantities may depend upon the order in which the quantities are being compared.
    - Example: In the mixture example above, we could also compare the ratio of the liters of syrup per liters of water, as shown:

Water (liters) x	3	6	9	12
Syrup (liters) y	1	2	3	4

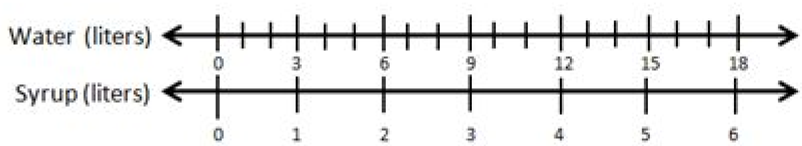
In this comparison, the ratio of the amount of syrup (y) to the amount of water (x) would be 1:3.

The graph of this relationship could be represented by:



Students should be aware of how the order in which quantities are compared affects the way in which the relationship is represented as a table of equivalent ratios or as a graph.

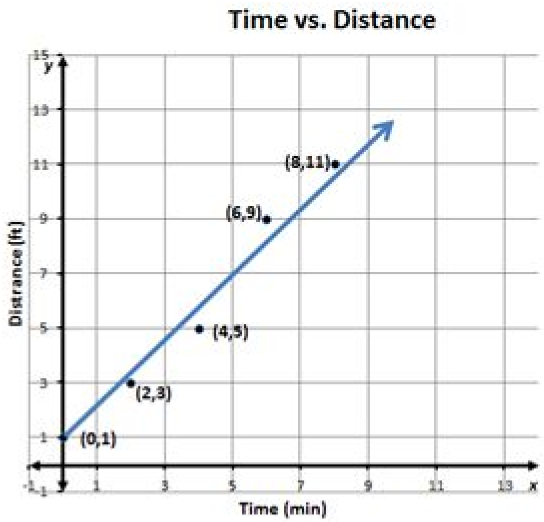
- Double number line diagrams can also be used to represent proportional relationships and create collections of pairs of equivalent ratios.
    - Example:



In this proportional relationship, there are three liters of water for each liter of syrup represented on the number lines.

- A graph representing a proportional relationship includes ordered pairs that lie in a straight line that, if extended, would pass through (0, 0), creating a pattern of horizontal and vertical increases. The context of the problem and the type of data being represented by the graph must be considered when determining whether the points are to be connected by a straight line on the graph.

- Example of the graph of a non-proportional relationship:



The relationship of distance (*y*) to time (*x*) is non-proportional. The ratio of *y* to *x* for each ordered pair is not equivalent. That is,

$$\frac{11}{8} \neq \frac{9}{6} \neq \frac{5}{4} \neq \frac{3}{2} \neq \frac{1}{0}$$

The points of the graph do not lie in a straight line. Additionally, the line does not pass through the point (0, 0), thus the relationship of *y* to *x* cannot be considered proportional.

- Practical situations that model proportional relationships can typically be represented by graphs in the first quadrant, since in most cases the values for *x* and *y* are not negative.
- Unit rates are not typically negative in practical situations involving proportional relationships.
- A unit rate could be used to find missing values in a ratio table.
  - Example: A store advertises a price of $25 for 5 DVDs.

What would be the cost to purchase 2 DVDs? 3 DVDs? 4 DVDs?

# DVDs	1	2	3	4	5
Cost	$5	?	?	?	$25

The ratio of $25 per 5 DVDs is also equivalent to a ratio of $5 per 1 DVD, which would be the unit rate for this relationship. This unit rate could be used to find the missing value of the ratio table above. If we multiply the number of DVDs by a constant multiplier of 5 (the unit rate) we obtain the total cost. Thus, 2 DVDs would cost $10, 3 DVDs would cost $15, and 4 DVDs would cost $20.

Vocabulary	Instructional Activities Organized by Learning Objective
ratio unit rate ratio table multiplicative relationships equivalent equivalent ratio proportion proportional relationship input/output tables double number line graphs verbal descriptions ordered pairs coordinate plane multiple representations constant	Textbook **Virginia Math Connects, Course 2**, ©2012, Price, et al, McGraw-Hill School Education Group 1: Unit Rates, page(s) 265- 271; Proportional, page(s) 272- 283; Scale Drawings, page(s) 284 – 291; Similar Figures, page(s) 293 – 303; Real World Application, page(s) 304 - 305 Notes 7.4/6.12 Proportions Interactive Notes Page Resources ● Print Virginia, Coach, New SOL Edition, Mathematics, Grade 6, Represent Proportional Relationships, page(s) 194 - 200

Assessment	Unit Rates and Proportional Relationships, pages(s) 201 - 205 ● Technology-based Studyjams.com, Ratios Studyjams.com, Proportions Studyjams.com, Rate Station Activities Proportions Proportions, Ratios, and Scale Drawings
Cross-Curricular Connections	Tiered Differentiations
Proportions assist students in being able to correctly convert between different units of measurement in science labs. For example, students can use proportions to convert within and between the U. S. Customary and the metric systems.	● Suggested manipulatives: linking cubes, centimeter cubes, toothpicks graph paper, number lines, ● Review SOL 6.1 prior to starting this unit. ● By using manipulatives, number lines, tables and graphs,students will build a conceptual understanding of proportional relationships and unit rates which will help them understand the more abstract representations and complex computations such as slope and intercepts.