

Richmond Public Schools
Curriculum Framework
Algebra II

Strand: Functions	
<p>All.7</p>	<p>The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include</p> <ul style="list-style-type: none"> a) domain, range, and continuity; b) intervals in which a function is increasing or decreasing; c) extrema; d) zeros; e) intercepts; f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; i) vertical and horizontal asymptotes; j) inverse of a function; and k) composition of functions, algebraically and graphically.
Suggested Pacing	
<p>A, C-G) 5 Class Periods B, H-K) 6 Class Periods</p>	
Spiraling Standards	
<p>8.14-The student will</p> <ul style="list-style-type: none"> a) evaluate an algebraic expression for given replacement values of the variables; and b) simplify algebraic expressions in one variable. <p>8.15-The student will</p> <ul style="list-style-type: none"> a) determine whether a given relation is a function; and b) determine the domain and range of a function. 	<p>AFDA.1-The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families and their characteristics. Key concepts include</p> <ul style="list-style-type: none"> a) domain and range; b) intervals on which a function is increasing or decreasing; c) absolute maxima and minima; d) zeros; e) intercepts;

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<p>8.17-The student will solve multistep linear equations in one variable with the variable on one or both sides of the equation, including practical problems that require the solution of a multistep linear equation in one variable.</p> <p>A.1-The student will</p> <p>b) evaluate algebraic expressions for given replacement values of the variables.</p> <p>A.7-The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically, including</p> <p>a) determining whether a relation is a function; b) domain and range; c) zeros; d) intercepts; e) values of a function for elements in its domain; and f) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs</p>	<p>f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; and i) vertical and horizontal asymptotes.</p> <p>MA.1-The student will investigate and identify the properties of polynomial, rational, piecewise, and step functions and sketch the graphs of the functions.</p> <p>MA.2-The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, solve equations, and solve practical problems.</p> <p>MA.3-The student will apply compositions of functions and inverses of functions to practical situations and investigate and verify the domain and range of resulting functions.</p> <p>MA.4-The student will determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity.</p> <p>MA.5-The student will investigate and describe the continuity of functions.</p>
Essential Questions	Common Misconceptions
<p>What is a function? What is the relationship between domain and range? What is the relationship between a function and its inverse? What is a zero of a function? What is meant by composition of functions?</p>	<p>Students may misidentify a relation as a function. Students may misidentify the domain as the range and vice versa Students may misidentify intercepts. Students may not recognize zeros as solutions or x-intercepts. Students may not accurately note the intervals in which the function is increasing or decreasing.</p>

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<p>What is the relationship between the degree of a function and the graph of a function? What is the relationship between exponential and logarithmic functions? How can the calculator be used to investigate the shape and behavior of polynomial, exponential, and logarithmic functions? What is an asymptote? What is meant by the end behavior of a function? What is meant by the turning points of a function and how are they found?</p>	<p>Asymptotes may be misidentified. Students may substitute the wrong equations when solving composition functions. Students may have trouble recognizing the inverse of functions graphically.</p>
<p>Understanding the Standard</p>	<p>Essential Knowledge and Skills</p>
<ul style="list-style-type: none"> • Functions may be used to model practical situations. • Functions describe the relationship between two variables where each input is paired to a unique output. • Function families consist of a parent function and all transformations of the parent function. • The domain of a function is the set of all possible values of the independent variable. • The range of a function is the set of all possible values of the dependent variable. • For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $(x, f(x))$ is a member of f. • A function is said to be continuous on an interval if its graph has no jumps or holes in that interval. 	<p>The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to</p> <ul style="list-style-type: none"> • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically, including graphs with discontinuities. (a, d, e) • Describe a function as continuous or discontinuous. (a) • Given the graph of a function, identify intervals on which the function (linear, quadratic, absolute value, square root, cube root, polynomial, exponential, and logarithmic) is increasing or decreasing. (b) • Identify the location and value of absolute maxima and absolute minima of a function over the domain of the function graphically or by using a graphing utility. (c) • Identify the location and value of relative maxima or relative minima of a function over some interval of the domain graphically or by using a graphing utility. (c)

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- The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by a function.
- Discontinuous domains and ranges include those with removable (holes) and nonremovable (asymptotes) discontinuities.
- A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function.
- A function, $f(x)$, is increasing over an interval if the values of $f(x)$ consistently increase over the interval as the x values increase.
- A function, $f(x)$, is decreasing over an interval if the values of $f(x)$ consistently decrease over the interval as the x values increase.
- A function, $f(x)$, is constant over an interval if the values of $f(x)$ remain constant over the interval as the x values increase.
- Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation. Examples may include:

Equation/Inequality	Set Notation	Interval Notation
$x = 3$	$\{3\}$	
$x = 3$ or $x = 5$	$\{3, 5\}$	
$0 \leq x < 3$	$\{x \mid 0 \leq x < 3\}$	$[0, 3)$
$y \geq 3$	$\{y \mid y \geq 3\}$	$[3, \infty)$
Empty (null) set \emptyset	$\{\}$	

- For any x value in the domain of f , determine $f(x)$. (f)
- Represent relations and functions using verbal descriptions, tables, equations, and graphs. Given one representation, represent the relation in another form. (g)
- Describe the end behavior of a function. (h)
- Determine the equations of vertical and horizontal asymptotes of functions (rational, exponential, and logarithmic). (i)
- Determine the inverse of a function (linear, quadratic, cubic, square root, and cube root). (j)
- Graph the inverse of a function as a reflection over the line $y = x$. (j)
- Determine the composition of two functions algebraically and graphically. (k)
- Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f, g, h, i, j, k)

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- A function, f , has an absolute maximum located at $x = a$ if $f(a)$ is the largest value of f over its domain.
- A function, f , has an absolute minimum located at $x = a$ if $f(a)$ is the smallest value of f over its domain.
- Relative maximum points occur where the function changes from increasing to decreasing.
- A function, f , has a relative maximum located at $x = a$ over some open interval of the domain if $f(a)$ is the largest value of f on the interval.
- Relative minimum points occur where the function changes from decreasing to increasing.
- A function, f , has a relative minimum located at $x = a$ over some open interval of the domain if $f(a)$ is the smallest value of f on the interval.
- A value x in the domain of f is an x -intercept or a zero of a function f if and only if $f(x) = 0$.
- Given a polynomial function $f(x)$, the following statements are equivalent for any real number, k , such that $f(k) = 0$:
 - k is a zero of the polynomial function $f(x)$ located at $(k, 0)$;
 - k is a solution or root of the polynomial equation $f(x) = 0$;
 - the point $(k, 0)$ is an x -intercept for the graph of $f(x) = 0$; and
 - $(x - k)$ is a factor of $f(x)$.
- Connections between multiple representations (graphs, tables, and equations) of a function can be made.

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<ul style="list-style-type: none"> • End behavior describes the values of a function as x approaches positive or negative infinity. • If (a, b) is an element of a function, then (b, a) is an element of the inverse of the function. • The reflection of a function over the line $y = x$ represents the inverse of the reflected function. • A function is invertible if its inverse is also a function. For an inverse of a function to be a function, the domain of the function may need to be restricted. • Exponential and logarithmic functions are inverses of each other. • Functions can be combined using composition of functions. • Two functions, $f(x)$ and $g(x)$, are inverses of each other if $f(g(x)) = g(f(x)) = x$. 	
Vocabulary	Instructional Activities Organized by Learning Objective
<p>function, linear function, quadratic function, absolute value function, square root function, cube root function, rational function, polynomial function, exponential function, logarithmic function, inverse of a function, composition of functions, function families, transformation, reflection, parent function, domain, range, input, output, independent variable, dependent variable, continuity, interval, extrema, maxima, minima, increasing, decreasing, intercept, values, end behavior, zeros, vertical asymptote, horizontal asymptote, restricted, discontinuous, continuous, constant, absolute, relative, set notation, interval notation</p>	<p>Textbook</p> <p>Eureka Math Precalculus and Advanced Topics Module 3 Eureka Math Algebra 2 Module 1 Topic B Lesson 15</p> <p>Algebra 2, ©2012, Price, et al, McGraw-Hill page(s) 417 – 423, 411,412</p> <p>Notes</p> <p>Functions and Relations (Spark Notes) Domain and Range (IntMath) Intervals (MathBits Notebook) Extrema (Cool Math)</p>
Assessment	
<p>Common Assessment AII.7a</p>	

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<p>Common Assessment AII.7b Common Assessment AII.7d Common Assessment AII.7e Common Assessment AII.7h Common Assessment AII.7i Common Assessment AII.7j Common Assessment AII.7k</p>	<p>How to Find Zeros of a Function (Analyze Math)</p> <p>Resources</p> <ul style="list-style-type: none"> • Print Coach book, Virginia edition, lesson 19 of chapter 3, lesson 31,32 of chapter 3 • Technology-based Domain and Range of Functions (Khan Academy) Continuity (Khan Academy) Increasing/Decreasing Intervals (Desmos) Zeros and Intercepts (Khan Academy) X- and Y-Intercepts (Khan Academy) Polynomial Pandemonium (Desmos) <p>Station Activities</p> <p>Any activity listed under other Algebra 2 objectives may be adjusted to address concepts in AII.7</p>
Cross-Curricular Connections	Tiered Differentiations
<p>Functions are represented in a wide range of fields. Challenge student to find applications, connections, and utility in Science (Chemistry, Physics) CTE (Engineering, Technology, Marketing, Programming)</p>	<p>Tier 1-Student will calculate values algebraically and graphically. Tier 2- Student will calculate values with the help of a calculator for verification. Tier 3-Students will be given graphs and a calculator to help in verifying answers calculated.</p>