Richmond Public Schools

Curriculum Framework

Algebra, Functions, and Data Analysis

Strand: Algebra Functions
AFDA.1 The student will investigate and analyze linear, quadratic, exponential, and logarithmic function families and their characteristics. Key concepts include **a) domain and range;** **b) intervals on which a function is increasing or decreasing;** **c) absolute maxima and minima;** **d) zeros;** **e) intercepts;** **f) values of a function for elements in its domain;** **g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs;** **h) end behavior; and** **i) vertical and horizontal asymptotes.**
Suggested Pacing
12 Class Periods

Spiraling Standards	
A.1-The student will b) evaluate algebraic expressions for given replacement values of the variables. AI.7-The student will investigate and analyze linear and quadratic function families and their characteristics both algebraically and graphically, including a) determining whether a relation is a function; b) domain and range; c) zeros; d) intercepts; e) values of a function for elements in its domain; and	MA.1-The student will investigate and identify the properties of polynomial, rational, piecewise, and step functions and sketch the graphs of the functions. MA.2-The student will investigate and identify the characteristics of exponential and logarithmic functions to graph the function, solve equations, and solve practical problems. MA.3-The student will apply compositions of functions and inverses of functions to practical situations and investigate and verify the domain and range of resulting functions.

f) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs All.7-The student will investigate and analyze linear, quadratic, absolute value, square root, cube root, rational, polynomial, exponential, and logarithmic function families algebraically and graphically. Key concepts include a) domain, range, and continuity; b) intervals in which a function is increasing or decreasing; c) extrema; d) zeros; e) intercepts; f) values of a function for elements in its domain; g) connections between and among multiple representations of functions using verbal descriptions, tables, equations, and graphs; h) end behavior; i) vertical and horizontal asymptotes; j) inverse of a function; and k) composition of functions algebraically and graphically.	MA.4-The student will determine the limit of an algebraic function, if it exists, as the variable approaches either a finite number or infinity. MA.5-The student will investigate and describe the continuity of functions.
Essential Questions	**Common Misconceptions**
What is a function? What is average rate of change and how can it be determined? When data is given in the form of a graph, table, rule, or words? How is it recognized as a linear, quadratic, exponential, or logarithmic function? How does a function representing a set of data correlate to the parent linear function? What is the best representation of a data set (table, graph, equation) given a real-world situation? What is the zero of a function?	Students may misidentify a relation as a function. Students may misidentify the domain as the range and vice versa Students may misidentify intercepts. Students may not recognize zeros as solutions or x-intercepts. Stunts may not accurately note the intervals in which the function is increasing or decreasing. Asymptotes may be misidentified. Students may substitute the wrong equations when solving composition functions.

What is the relationship between domain and range? In a real-world problem, what is the difference between the domain of a function and the practical domain of the model? In a real-world problem, what is the difference between the range of a function and the practical range of the model? How does the leading coefficient determine the end behavior of a linear, quadratic, exponential, or logarithmic function? How can the graphing calculator be used to investigate linear, quadratic, exponential, or logarithmic functions?	Students may have trouble recognizing the inverse of functions graphically.
Understanding the Standard	**Essential Knowledge and Skills**
• A relation is a function if and only if each element in the domain is paired with a unique element of the range. • Functions are used to model practical phenomena. • Functions describe the relationship between two variables where each input is paired to a unique output. • Function families consist of a parent function and all transformations of the parent function. • The domain of a function is the set of all possible values of the independent variable. • The range of a function is the set of all possible values of the dependent variable. • For each x in the domain of f, x is a member of the input of the function f, $f(x)$ is a member of the output of f, and the ordered pair $(x, f(x))$ is a member of f. • The domain of a function may be restricted algebraically, graphically, or by the practical situation modeled by the function.	**The student will use problem solving, mathematical communication, mathematical reasoning, connections, and representations to** • Identify the domain, range, zeros, and intercepts of a function presented algebraically or graphically. Domains may be limited by problem context or in graphical representations. (a, d, e) • Identify intervals on which the function is increasing or decreasing. (b) • Identify the location and value of the absolute maximum and absolute minimum of a function over the domain of the function graphically or by using a graphing utility. (c) • For any x value in the domain of f, determine $f(x)$. (f) • Represent relations and functions using verbal descriptions, tables, equations, and graphs. Given one representation, represent the relation in another form. (g) • Detect patterns in data and represent arithmetic and geometric patterns algebraically. (g)

- A function can be described on an interval as increasing, decreasing, or constant over a specified interval or over the entire domain of the function.
- A function, *f(x)*, is increasing over an interval if the values of *f(x)* consistently increase over the interval as the *x* values increase.
- A function, *f(x)*, is decreasing over an interval if the values of *f(x)* consistently decrease over the interval as the *x* values increase.
- A function, *f(x)*, is constant over an interval if the values of *f(x)* remain constant over the interval as the *x* values increase.
- A function, *f*, has a maximum at $x = a$ if *f(a)* is the largest value of *f* over its domain.
- A function, *f*, has a minimum in some interval at $x = a$ if *f(a)* is the smallest value of *f* over its domain.
- Solutions and intervals may be expressed in different formats, including set notation, using equations and inequalities, or interval notation.
  - Examples may include:

Equation/ Inequality	Set Notation	Interval N
$x = 3$	{3}	
$x = 3$ or $x = 5$	{3, 5}	
$0 \leq x < 3$	$\{x \mid 0 \leq x < 3\}$	[0, 3)
$y \geq 3$	$\{y: y \geq 3\}$	[3, ∞)
Empty (null) set ∅	{ }	

- A value *x* in the domain of *f* is an *x*-intercept or a zero of a function *f* if and only if $f(x) = 0$.

- Describe the end behavior of a function. (h)
- Determine the equations of the horizontal asymptote of an exponential function and the vertical asymptote of a logarithmic function. (i)
- Investigate and analyze characteristics and multiple representations of functions with a graphing utility. (a, b, c, d, e, f, g, h, i)

• The *x*-intercept is the point at which the graph of a relation or function intersects with the *x*-axis. It can be expressed as a value or a coordinate. • The *y*-intercept is the point at which the graph of a relation or function intersects with the *y*-axis. It can be expressed as a value or a coordinate. • Given a polynomial function *f(x)*, the following statements are equivalent for any real number, *k*, such that *f(k)* = 0: *k* is a zero of the polynomial function *f(x)* located at (*k*, 0); *k* is a solution or root of the polynomial equation *f*(*x*) = 0; the point (*k*, 0) is an *x*-intercept for the graph of polynomial *f(x)* = 0; and *(x – k)* is a factor of polynomial *f(x)*. • Connections between multiple representations (graphs, tables, and equations) of a function can be made. • In an arithmetic pattern, the common difference is the value that is added to obtain the next value in the pattern. • In geometric number patterns, the common ratio is a multiplier used to obtain the next value in the pattern. • End behavior describes a function's values as *x* approaches positive or negative infinity. • Asymptotes can be used to describe local behavior and end behavior of graphs. They are lines or other curves that approximate the graphical behavior of a function.	
Vocabulary	
function, linear function, quadratic function, absolute value function, square root function, cube root function, rational function,	**Textbook** Eureka Math Precalculus and Advanced Topics Module 3

polynomial function, exponential function, logarithmic function, inverse of a function, composition of functions, function families, transformation, reflection, parent function, domain, range, input, output, independent variable, dependent variable, continuity, interval, extrema, maxima, minima, increasing, decreasing, intercept, values, end behavior, zeros, vertical asymptote, horizontal asymptote, restricted, discontinuous, continuous, constant, absolute, relative, set notation, interval notation	Eureka Math Algebra 2 Module 1 Topic B Lesson 15 ● Algebra 1, ©2012, Price, et al, McGraw-Hill School Education Group, page(s) 38- 53, 153-160, 525-536 ● Algebra 2, ©2012, Price, et al, McGraw-Hill School Education Group, page(s) 360- 362, 383-397, 570-574, 577-580 ● Algebra, Functions, and Data Analysis, A Virginia Course, The Consortium for Foundation Mathematics page(s) 43-51, 165-284, 291-298, 399-513, 519-526, 527-683, 689-699
Assessment	**Notes** Functions and Relations (Spark Notes) Domain and Range (IntMath) Intervals (MathBits Notebook) Extrema (Cool Math) How to Find Zeros of a Function (Analyze Math)
Classroom Teacher Developed Assessments	**Resources** ● **Print** Coach book Algebra 1, Virginia edition, Lesson(s) 7, 9-11, 23-28 page(s) 45-48, 58-71, 154-196 Coach book Algebra 2, Virginia edition, Lesson(s) 33 page(s) 213-219 ● **Technology-based** Domain and Range of Functions (Khan Academy) Continuity (Khan Academy) Increasing/Decreasing Intervals (Desmos) Zeros and Intercepts (Khan Academy) X- and Y-Intercepts (Khan Academy)

	Polynomial Pandemonium (Desmos) **Station Activities** Relations, Functions, Domain & Range Task Cards Evaluating Functions Scavenger Hunt Exponential and Logarithmic Functions Stations Activity Linear Equations Stations Quadratic Functions Lab
Cross-Curricular Connections	Tiered Differentiations
Functions are represented in a wide range of fields. Challenge student to find applications, connections, and utility in Science (Chemistry, Physics) CTE (Engineering, Technology, Marketing, Programming)	**Tier 1**-Student will calculate values algebraically and graphically. **Tier 2**- Student will calculate values with the help of a calculator for verification. **Tier 3**-Students will be given graphs and a calculator to help in verifying answers calculated.